On Strain Energy Release Rates for Interfacial Cracks

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The bimaterial constant ε is necessarily used in the interfacial crack problems. Some authors tried to neglect the effects of the bimaterial constant ε. To investigate the effects of the bimaterial constant ε, the individual strain energy release rates, $G_I{}^*$ and $G_{II}{}^*$, which are obtained by neglecting the bimaterial constant ε, are examined. Three examples were investigated to see the importance of the effects of the bimaterial constant ε. Firstly, the analytical results of a center interfacial crack between two dissimilar materials in an infinite plate are illustrated for various loading conditions. The phase angles of a center interfacial crack are also examined to check the importance of the bimaterial constant ε. Secondly, the individual energy release rates of a center crack paralleling an interface are examined. Thirdly, the finite element results of a four-point bending beam with two symmetrical cracks paralleling an interface are illustrated. Considering the analytical and numerical results, we can see that the bimaterial constant ε is an important factor in the interfacial crack problem, which can not be neglected.

Key Words: Interfacial Crack, Phase Angle, Bimaterial Constant ε, Energy Release Rate

1. Introduction

The interfacial crack problem is very important in the fracture of advanced composite materials, ranging from delamination to interfacial debonding of their constituents.

In the late 1950's, Williams(1959) solved the interfacial crack problem by using an eigenfunction technique for the first time. He demonstrated rapid oscillating stresses near the crack tip. Erdogan(1963) solved this problem by using complex function theory. He found the solution of a homogeneous Hilbert problem and obtained quantitative expressions for stresses. He pointed out that the phenomenon of stress oscillation may be ignored because the oscillating region is very small. England (1965) claimed that the cause of oscillating phenomenon is the use of both an idealized physical model and the classical linear theory of elasticity. He also pointed out that the oscillating region is very small. Rice and Sih(1965) showed how the complex-variable method combined with eigenfunction expansion can be applied to formulate the problem of bonded dissimilar elastic planes containing cracks along the bond.

In order to overcome the physically inadmissible phenomena, many authors presented their own models. Comninou(1977, 1978) and Comninou and Schmueser(1979) solved the interfacial crack problem by assuming that there is a small contact zone near the crack tips. Atkin son (1977) presented a model in which a third material was used between two different elastic media. Mak (Mak, Keer, Chen and Lewis, 1980) presented a no-slip interfacial crack model to overcome relative shear slip within the crack Sinclair(1980) presented a model which preserves

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the interface, allows a crack-opening mode and is free from inter-penetration.

Although some authors succeeded to solve the physically inadmissible phenomena, we do not yet know which model is right for describing the behavior of the real interfacial crack. The crack tip stresses depend on each model. According to the model used. K_I and K_{II} are different for the same problem. Therefore, it is better not to use stress intensity factors for the interfacial crack until the crack tip stress field is clearly defined.

Suo and Hutchinson(1989) tried to neglect the effects of the bimaterial constant ε in view of the clarification in interpretation and simplification in approach. However, it seems dangerous to neglect the effects of the bimaterial constant ε in a real interfacial crack problem.

The total strain energy release rate, which is well defined, has been used as a fracture parameter for the interfacial crack propagation. If we use only the total energy release rate as a fracture parameter, some problems arise. For example, for an infinite plate with an interfacial crack under tensile or shear loading, the total energy release rates for both loading conditions are same. The fracture resistance for the shear loading condition is usually higher than that for the tensile loading condition. Therefore, it is necessary to obtain the individual energy release rate to get a better fracture criterion for the interfacial crack. Sun and Jih(1987) obtained the explicit form for the individual energy release rates by using the crack closure integral. However, G_I and G_{II} depend on the crack increment, the nature of which is still an open problem to debate.

In this study, the individual energy release rates G_I^* and G_{II}^*, which are obtained by neglecting the effects of the bimaterial constant ε, are examined. To check the importance of the bimaterial constant ε, the analytical and finite element results are illustrated for various kinds of loading and geometric conditions. The phase angles will be also examined to check the effects of the bimaterial constant ε.

2. Basic Equations

In the case of a center interfacial crack located between two dissimilar semi-infinite plates which are subjected to the mixed loading condition as shown in Fig. 1, where $(\varepsilon_{11}^{\infty})_1=(\varepsilon_{11}^{\infty})_2$, the stress function can be obtained by the Hilbert formulation as follows (Rice and Sih, 1965; Sun and Jih, 1987):

$$\Phi_1(z)=\frac{\sigma_{22}^{\infty}-i\sigma_{12}^{\infty}}{1+e^{2\pi\varepsilon}}(z-2i\varepsilon a)(z+a)^{-1/2}(z-a)^{-1/2}\left[\frac{z+a}{z-a}\right]^{i\varepsilon}, \tag{1}$$

where

$$\varepsilon=\frac{1}{2\pi}\ ln\left[\left[\frac{\kappa_1}{\mu_1}+\frac{1}{\mu_2}\right]/\left[\frac{\kappa_2}{\mu_2}+\frac{1}{\mu_1}\right]\right], \tag{2}$$

$$\kappa_j=\begin{cases}3-4\nu_j & \text{for plane strain,}\\ (3-\nu_j)/(1+\nu_j) & \text{for plane stress,}\end{cases} \tag{3}$$

$$\mu_j=\text{shear modulus.} \tag{4}$$

Rice and Sih(1965) defined the stress intensity factor $k=k_1-ik_2$ as follows,

$$k_1-ik_2=2\sqrt{2}e^{\pi\varepsilon}\lim_{z\to a}(z-a)^{1/2+i\varepsilon}\Phi_1(z). \tag{5}$$

From Eqs. (1) and (5), the stress intensity factors at $z=a$ are obtained as follows (Rice and Sih, 1965),

$$k_1=\frac{a^{1/2}}{\cosh\pi\varepsilon}[\sigma_{22}^{\infty}\{\cos(\varepsilon ln2a)+2\varepsilon\sin(\varepsilon ln2a)\}+\sigma_{12}^{\infty}\{\sin(\varepsilon ln2a)-2\varepsilon\cos(\varepsilon ln2a)\}], \tag{6}$$

$$k_2=\frac{a^{1/2}}{\cosh\pi\varepsilon}[\sigma_{12}^{\infty}\{\cos(\varepsilon ln2a)+2\varepsilon\sin(\varepsilon ln2a)\}-\sigma_{22}^{\infty}\{\sin(\varepsilon ln2a)-2\varepsilon\cos(\varepsilon ln2a)\}]. \tag{7}$$

In the above definitions, the stress intensity factors become functions of the measuring unit of the crack length because the ln terms have a dimension of length. The normal and shear stresses of the singular field acting on the interface a distance r ahead of the crack tip can be expressed in terms of k_1 and k_2 as follows:

$$\sqrt{2\pi r}\ \sigma_{yy}=K_0\cos(\varepsilon\ lnr+\psi), \tag{8}$$

$$\sqrt{2\pi r}\ \sigma_{xy}=K_0\sin(\varepsilon\ lnr+\psi), \tag{9}$$

where

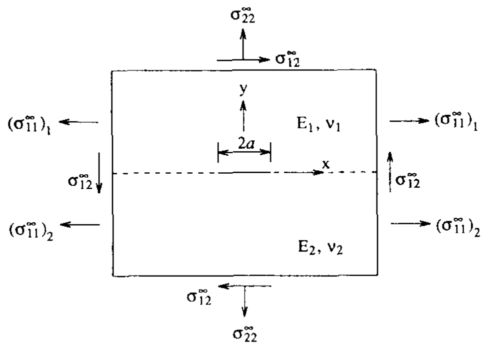

Fig. 1 Infinite plate with an interface crack subjected to stresses at infinity

$$K_0=\sqrt{\pi}\ \cosh(\pi\varepsilon)\ \sqrt{k_1^2+k_2^2}, \tag{10}$$
$$\psi=\tan^{-1}(k_2/k_1), \tag{11}$$

Rice(1988) expressed the relative displacement of two points on the top and bottom crack surfaces in terms of a complex stress intensity factor k, which was introduced by Hutchinson et al. (Hutchinson Mear and Rice, 1987).

$$\Delta u_y+i\Delta u_x=\left[\frac{\chi_1+1}{\mu_1}+\frac{\chi_2+1}{\mu_2}\right]\frac{Kr^{i\varepsilon}\sqrt{r}}{2\sqrt{2\pi}(1+2i\varepsilon)\cosh(\pi\varepsilon)}, \tag{12}$$

where

$$K=(k_1+ik_2)\sqrt{\pi}\ \cosh(\pi\varepsilon), \tag{13}$$

and k_1+ik_2 is the complex stress intensity factor as originally introduced by Rice and Sih(1965).

Matos et al. 1989) introduced the parameters K_I^* and K_{II}^* while presenting the crack surface displacement(CSD) method.

$$\Delta u_y+i\Delta u_x=\left[\frac{\chi_1+1}{\mu_1}+\frac{\chi_2+1}{\mu_2}\right]\frac{(K_I^*+iK_{II}^*)r^{i\varepsilon}\sqrt{r}}{2\sqrt{2\pi}(1+2i\varepsilon)\cosh(\pi\varepsilon)}. \tag{14}$$

Sun and Jih(1987) introduced the stress intensity factors K_I and K_{II} to remove the ambiguity of dimension as follows:

$$K_I-iK_{II}=2\sqrt{2\pi}\ e^{\pi\varepsilon}\lim_{z-a}(z-a)^{1/2}\left[\frac{z+a}{z-a}\right]^{i\varepsilon}\Phi_1(z). \tag{15}$$

The relative displacement of crack surface can be expressed in terms of K_I and K_{II}.

$$\Delta u_y+i\Delta u_x=\left[\frac{\chi_1+1}{\mu_1}+\frac{\chi_2+1}{\mu_2}\right]\frac{(K_I+iK_{II})(r/2a)^{i\varepsilon}\sqrt{r}}{2\sqrt{2\pi}\ (1+2i\varepsilon)}. \tag{16}$$

From Eqs. (1) and (15), Sun and Jih obtained the stress intensity factors K_I and K_{II} of a center interfacial crack.

$$K_I=\sqrt{\pi a}\ [\sigma_{22}^{\infty}-2\varepsilon\sigma_{12}^{\infty}]/\cosh(\pi\varepsilon), \tag{17}$$
$$K_{II}=\sqrt{\pi a}\ [\sigma_{12}^{\infty}+2\varepsilon\sigma_{22}^{\infty}]/\cosh(\pi\varepsilon). \tag{18}$$

The normal and shear stresses can be expressed in terms of K_I and K_{II} as follows:

$$\sqrt{2\pi r}\ \sigma_{yy}=K_0^*\ \cos(\varepsilon\ ln\frac{r}{2a}+\psi^*), \tag{19}$$
$$\sqrt{2\pi r}\ \sigma_{xy}=K_0^*\ \sin(\varepsilon\ ln\frac{r}{2a}+\psi^*), \tag{20}$$

where

$$K_0^*=\cosh(\pi\varepsilon)\ \sqrt{K_I^2+K_{II}^2}, \tag{21}$$
$$\psi^*=\tan^{-1}(K_{II}/K_I), \tag{22}$$

From Eqs. (6, 7) and (17, 18), we can obtain the relationship between K_I, K_{II} and k_1, k_2 as follows:

$$k_1=[K_I\ \cos(\varepsilon ln2a)+K_{II}\ \sin(\varepsilon ln2a)]/\sqrt{\pi} \tag{23}$$
$$k_2=[K_{II}\ \cos(\varepsilon ln2a)-K_I\ \sin(\varepsilon ln2a)]/\sqrt{\pi} \tag{24}$$

As we can see in the above equations, many authors define the stress intensity factors differently. The meaning of stress intensity factors for the interfacial crack is not clear yet.

Sun and Jih(1987) obtained the total and individual energy release rates through the crack closure integral

$$G_I=\frac{1}{2}G+\lim_{\delta a-0}\frac{\cosh(\pi\varepsilon)}{8(1+4\varepsilon^2)\pi}\left[\frac{\chi_1+1}{\mu_2}+\frac{\chi_2+1}{\mu_1}\right]\left[1+\frac{\delta a}{2a}+\cdots\right][B(K_I^2-K_{II}^2)-2CK_IK_{II}], \tag{25}$$

$$G_{II}=\frac{1}{2}G-\lim_{\delta a-0}\frac{\cosh(\pi\varepsilon)}{8(1+4\varepsilon^2)\pi}\left[\frac{\chi_1+1}{\mu_1}+\frac{\chi_2+1}{\mu_2}\right]\left[1+\frac{\delta a}{2a}+\cdots\right][B(K_I^2-K_{II}^2)-2CK_IK_{II}], \tag{26}$$

$$G = G_I + G_{II} = \frac{1}{16}\left[\frac{\chi_1+1}{\mu_1} + \frac{\chi_2+1}{\mu_2}\right](K_I^2 + K_{II}^2), \quad (27)$$

where

$$B = Re[A], \quad (28)$$
$$C = -Im[A], \quad (29)$$
$$A = \frac{\sqrt{\pi}}{2}\left[\frac{1}{2} + i\varepsilon\right]\left[\frac{\delta a}{4a}\right]^{-2i\varepsilon} \Gamma\left[\frac{1}{2} - i\varepsilon\right] / \Gamma\left[1 - i\varepsilon\right], \quad (30)$$

and δa and Γ are crack increment and Gamma function, respectively. The total energy release rate is well defined. However, G_I and G_{II} as $\delta a \to 0$ do not exist. This kind of phenomenon is not solved yet.

Parameter ε is an important factor, which always appears in the interfacial crack problem. If we follow Ref. 11 (Suo and Hutchinson, 1989) and neglect the effects of the bimaterial constant ε, the individual energy release rates can be also simplified. If we introduce G_I^* and G_{II}^*, which are obtained by neglecting the effects of the bimaterial constant ε, they can be expressed from Eqs. (25) and (26) as follows :

$$G_I^* = \frac{1}{16}\left[\frac{\chi_1+1}{\mu_1} + \frac{\chi_2+1}{\mu_2}\right] K_I^2, \quad (31)$$
$$G_{II}^* = \frac{1}{16}\left[\frac{\chi_1+1}{\mu_1} + \frac{\chi_2+1}{\mu_2}\right] K_{II}^2. \quad (32)$$

The summation of G_I^* and G_{II}^* is exactly same as that of G_I and G_{II}.

3. Numerical Examples and Discussion

To see the dependence of energy release rates on $\delta a/a$, material property, loading condition, and geometry, three examples were investigated.

The first example considered is a center interfacial crack between two dissimilar materials in an infinite plate subjected to stresses at infinity as shown in Fig. 1. This problem was originally solved by Rice and Sih(1965). Using Eqs. (17, 18), (25~32), we can calculate G_I, G_{II}, G_I^*, G_{II}^*, and G.

Parameters p and q were introduced to investigate the dependence of energy release rates on the material properties.

$$p = E_1/E_2, \quad (33)$$
$$q = \nu_1/\nu_2, \quad (34)$$

The range used is from 0.01 to 1 for p and from 0.1 to 1 for q. E_1 and ν_2 were set to 1 and 0.3, respectively. From Eq. (2), ε (p, q) can be obtained as shown in Table 1.

Figures 2~4 show the dependence of energy release rates on $\delta a/a$ under tensile load for three types of material. The energy release rates were normalized with respect to $(\sigma_{22}^{\infty})^2 \pi a/E_1$. From Figs. 2~4, we can see that the dependence of energy release rates on $\delta a/a$ becomes low as p increases. Similarly, the dependence of energy release rates on $\delta a/a$ becomes low as q increases. If bimaterial constant ε is very small and the bi-material structure is under the pure tension, the

Table 1 Values of $\varepsilon(p, q)$

$\varepsilon(p, q)$	$p=0.01$	$p=0.1$	$p=1$
$q=0.1$	0.1651	0.1383	0.0215
$q=0.5$	0.1415	0.1180	0.0119
$q=1$	0.1138	0.0938	0

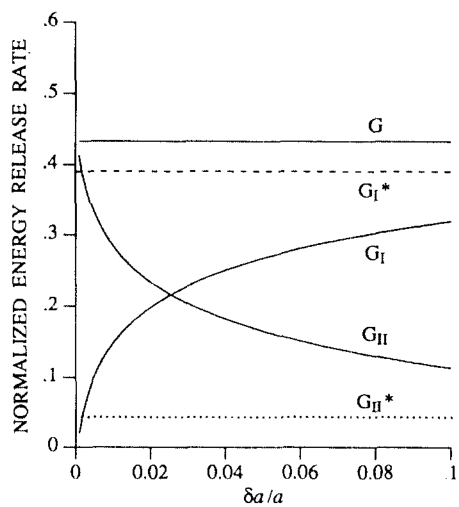

Fig. 2 Dependence of energy release rates on $\delta a/a$ under tension ($p=0.01$, $q=0.1$)

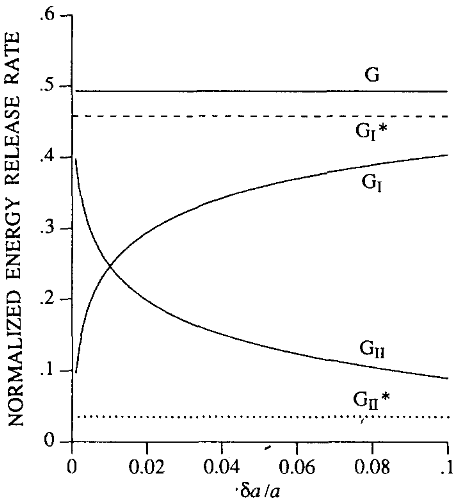

Fig. 3 Dependence of energy release rates on $\delta a/a$ under tension (p=0.1, q=0.1)

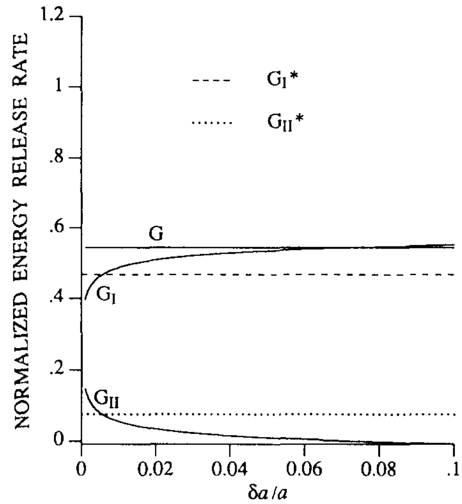

Fig. 5 Dependence of energy release rates on $\delta a/a$ under mixed load (p=0.1, q=1, γ=0.2)

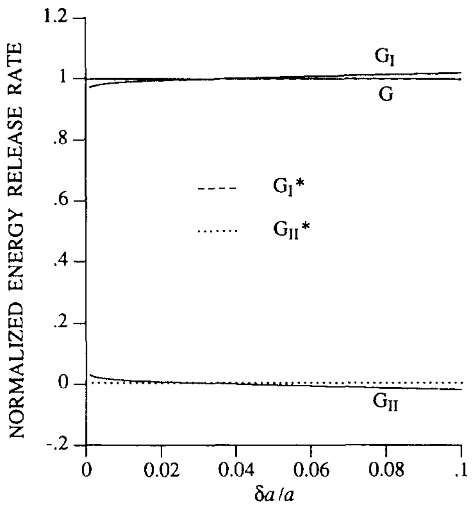

Fig. 4 Dependence of energy release rates on $\delta a/a$ under tension (p=1, q=0.1)

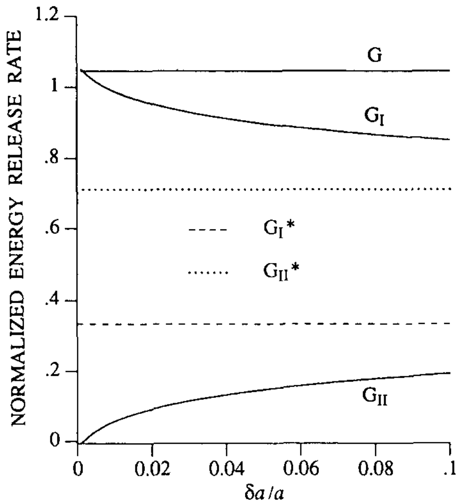

Fig. 6 Dependence of energy release rates on $\delta a/a$ under mixed load (p=0.1, q=1, γ=1)

dependence of G_I and G_{II} on $\delta a/a$ is low (see Fig. 4) and G_I* and G_{II}* almost coincide with G_I and G_{II}, respectively. Thus, the effects of the bimaterial constant seem to be negligible when the bi-material structure with small ε is under the pure tension.

Figures 5 and 6 show the dependence of energy release rates on $\delta a/a$ under mixed load for p=0. 1 and q=1. Here γ is the ratio, $\sigma_{12}^{\infty}/\sigma_{22}^{\infty}$. When the shear component of loading is about 0.7, G_I* and G_{II}* have similar values. As the shear component of loading increases, the position of G_I* and G_{II}* change and the curved shapes of G_I and G_{II} change.

Figure 7 shows the dependence of energy release rates on $\delta a/a$ under mixed load for $p=1$, $q=0.5$, and $\gamma=1$. In this case, the bimaterial constant ε is very small. We can see that $G_I{}^*$ and $G_{II}{}^*$ do not coincide respectively with G_I and G_{II} even for a very small ε. Thus, if the bi-material structure is under mixed load, the effects of the bimaterial constant ε can not be neglected even for a very small ε.

To check the importance of the effects of the bimaterial constant ε, we will examine the phase angles of a center interfacial crack. In the case of a center interfacial crack located between two dissimilar semi-infinite plates which are subjected to the mixed loading condition, ψ^* and ψ can be obtained from Eqs. (11), (17, 18), (22~24) as follows :

$$\psi^*=\tan^{-1}\left[\frac{\sigma_{12}^{\infty}+2\varepsilon\sigma_{22}^{\infty}}{\sigma_{22}^{\infty}-2\varepsilon\sigma_{12}^{\infty}}\right], \tag{35}$$

$$\psi=\psi^*-\varepsilon\ ln2a. \tag{36}$$

As we can see in the above equations, ψ^* is well defined. But ψ is a function of the measuring unit of the crack length. If we neglect the effects of the bimaterial constant ε in the above equations, ψ^* and ψ are equal to the phase angle for the homogeneous case. The stress fields for the interfacial crack are also equal to those for the homogeneous case. Then, there are no special properties for the interfacial crack problem. In fact, the bimaterial constant ε for Graphite/Epoxy (typical advanced composite material) system is not so small ($\varepsilon=0.10354$). Figure 8 shows the relationship between phase angles and shear loading ratio for $p=0.1$, $q=1$, and $a=1$ ($\varepsilon=0.0938$). There is a difference between ψ and ψ^* by $\varepsilon ln2a$. Therefore, if we

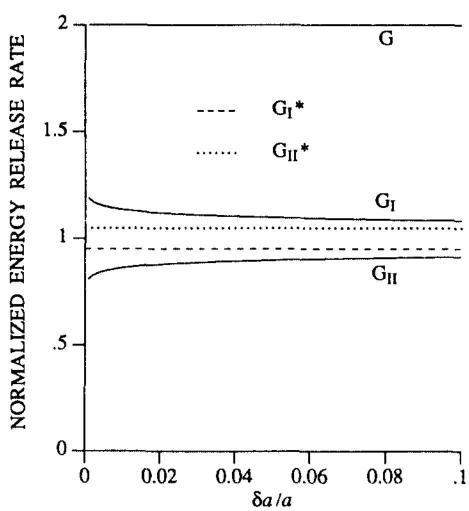

Fig. 7 Dependence of energy release rates on $\delta a/a$ under mixed load ($p=1$, $q=0.5$, $\gamma=1$)

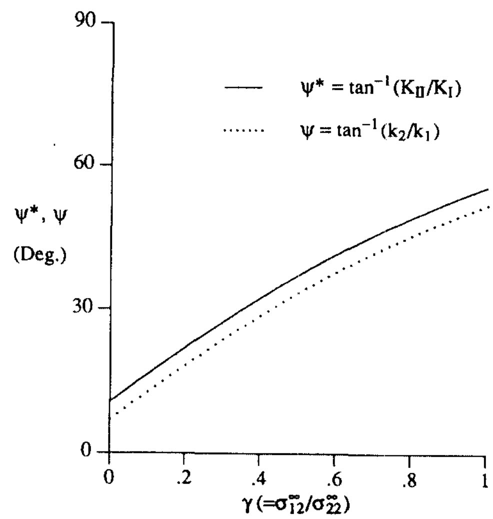

Fig. 8 Relationship between phase angles and shear loading ratio ($p=0.1$, $q=1$)

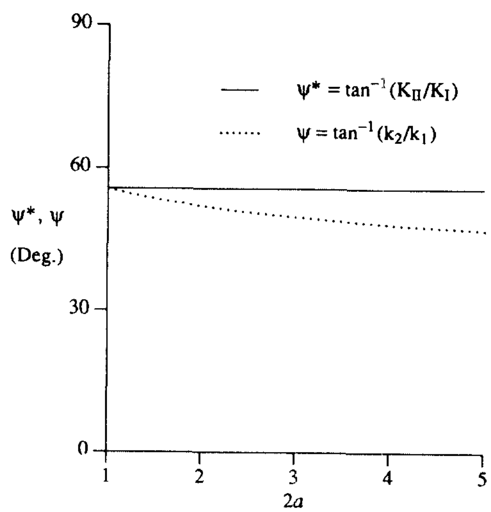

Fig. 9 Relationship between phase angles and crack length ($p=0.1$, $q=1$, $\gamma=1$)

define the stress intensity factors differently, the different phase angles will be obtained. The larger the bimaterial constant ε is, the larger the difference between ψ and ψ^* is. Figure 9 shows the relationship between phase angles and crack length. ψ^* is constant regardless of crack length. But ψ depends on crack length a. As the crack length increases, the phase angle ψ decreases and the difference between ψ and ψ^* increases. Thus, the effects of the bimaterial constant ε can not be neglected in the interfacial crack problem. It is not adequate to use ψ, which is based on k_1 and k_2, to characterize the interfacial crack properties because ψ depends on the measuring unit of crack length.

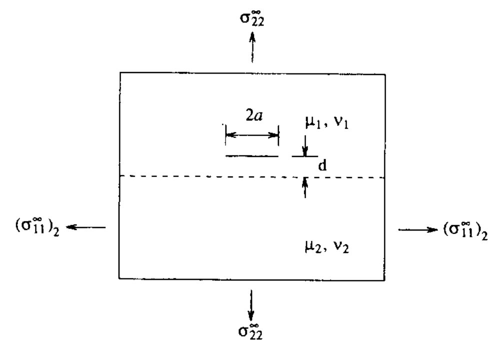

Fig. 10 Infinite plate with a crack paralleling an interface

The second example considered is a center crack paralleling an interface as shown in Fig. 10. This problem was solved by Isida and Noguchi(1983). They used the body force method to obtain the stress intensity factors. Employing the stress intensity factors for $\mu_2/\mu_1=4$ (see Table 6 of Ref. 16 (Isida and Noguchi, 1983)), the energy release rates were calculated and shown in Fig. 11. The energy release rates were normalized with respect to $(\sigma_{22}^{\infty})^2\pi\ a/E_1$. In the case of a center crack paralleling an interface, as the distance d decreases, G, G_I, and G_{II} will converge to those for the interfacial crack. Here G, G_I, and G_{II} for a center crack paralleling an interface have physical meaning because the crack is located in a homogeneous body. When the distance d is equasl to zero, this problem is reduced to the interfacial crack problem. Using the previous analytical solution, the energy release rates for this interfacial crack are shown in Fig. 12. Comparing with Fig. 11, G, G_I and G_{II} for five percent of $\delta a/a$ are almost similar to those for an asymptotic subinterface crack. Because the structure is under tensile load and $\varepsilon(=0.0678)$ is not so big, G_I^* and G_{II}^* in Fig. 12 are not much different from G_I and G_{II} for an asymptotic

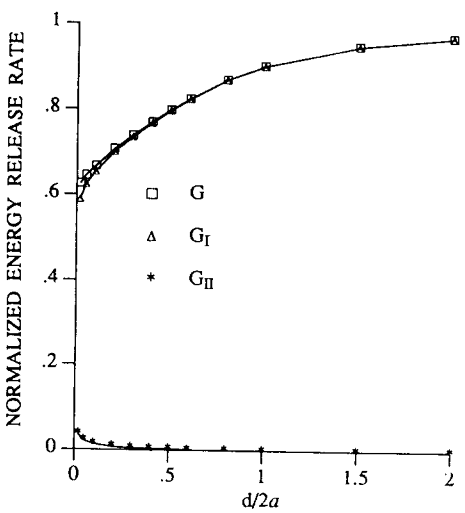

Fig. 11 Relationship between the energy release rates and the distance

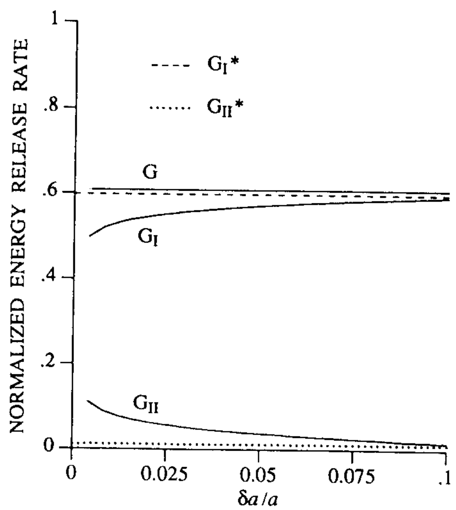

Fig. 12 Dependence of energy release rates on $\delta a/a$ under tension

subinterface crack in Fig. 11.

The third example considered is a four-point bending beam with two symmetrical cracks paralleling an interface as shown in Fig. 13. The four-point bending problem without a third material was solved by Matos et al. (Matos, McMeeking, Charalambides and Drory, 1989). They used the crack surface displacement (CSD) method and energy method to calculate the total strain energy release rate and individual stress intensity factors defined in Eq. (14).

ANSYS 4.4A was used to accomplish the stress analysis. The crack closure method was used to obtain the energy release rates. The total number of elements used in the finite element analysis was 1400 and an eight-noded isoparametric element was used. E_2/E_1 and E_3/E_1 are 10 and 0.3, respectively. a/l and h/l are 1.25 and 0.6, respectively, and ν_1, ν_2, and ν_3 are all 0.3.

Figure 14 shows the relationship between the energy release rates and the thickness of a third material. Here G, G_I, and $_{II}$ for a crack paralleling an interface have physical meaning because the crack is located in a homogeneous body. As the thickness of a third material decreases, G, G_I, and G_{II} will converge to the energy release rates for the interfacial crack. When the thickness of a third material is equal to zero, the problem is reduced to the interfacial crack problem. Using finite element analysis, the energy release rates for this interfacial crack were calculated and shown in Fig. 15. Comparing Fig. 14, G, G_I, and G_{II} for about five percent of $\delta a/a$ are almost similar to those for an asymptotic subinterface crack. However, G_I* and G_{II}* are very different from G_I and G_{II} for an asymptotic subinterface crack shown in Fig. 14. G_I* and G_{II}* were obtained from Ref. 15 (Matos, McMeeking, Charalambides, and Drory, 1989). We can see that the effects of the bimaterial constant ε can not be neglected for a four-point bending beam with interfacial crack.

4. Conclusions

The individual strain energy release rates G_I*

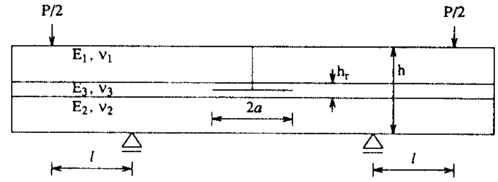

Fig. 13 The four-point bending beam with cracks paralleling an interface

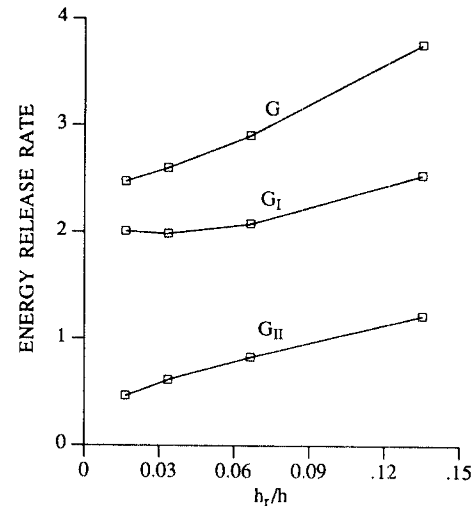

Fig. 14 Relationship between energy release rates and h_r for four point bending beam

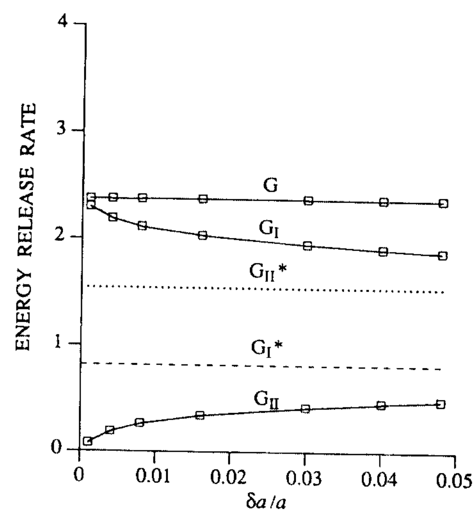

Fig. 15 Dependence of energy release rates on $\delta a/a$ for four point bending beam

and G_{II}^*, which are obtained by neglecting the effects of the bimaterial constant ε, are well defined. The summation of G_I^* and G_{II}^* is equal to the summation of G_I and G_{II}. However, care must be taken in using G_I^* and G_{II}^* because these values strongly depend on bimaterial constant ε.

If bimaterial constant ε is less than a few hundredths and the bi-material structure is under the pure tension or shear, the dependence of G_I and G_{II} on the crack increment is low. Then, G_I^* and G_{II}^* almost coincide with G_I and G_{II}, respectively. In this case, the effects of the bimaterial constant ε seem to be negligible and G_I^* and G_{II}^* can be used as individual energy release rates.

If bimaterial constant ε is not small, the dependence of G_I and G_{II} on the crack increment is very strong and G_I^* and G_{II}^* can not be used.

When the bi-material structure is under mixed load, G_I^* and G_{II}^* change their position as the shear component of loading increases. In the case of mixed loading condition, even though the bimaterial constant ε is very small, G_I^* and G_{II}^* are different from G_I and G_{II}. Thus, G_I^* and G_{II}^* can not be used in this case even if ε is small.

Therefore, bimaterial constant ε is an important factor in the interfacial crack problem, which can not be neglected.

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